**Method for Inversion of CIECAM97s**

(Provided to TC1-34 by R.W.G. Hunt, September, 1997)

Starting Data:
- Q or J, M or C, H or h
- \(A_w, n, z, F_L, N_{bb}, N_{cb}\) Obtained Using Forward Model

Surround Parameters: \(F, c, F_L, N_c\)

Luminance Level Parameters: \(L_A, D\)

Unique Hue Data:
- Red: \(h = 20.14, e = 0.8\)
- Yellow: \(h = 90.00, e = 0.7\)
- Green: \(h = 164.25, e = 1.0\)
- Blue: \(h = 237.53, e = 1.2\)

(1) From Q Obtain J (if necessary)

\[ J = 100(Qc/1.24)^{1/0.67} / (A_w + 3)^{0.9/0.67} \]

(2) From J Obtain A

\[ A = (J/100)^{1/cz} A_w \]

(3) Using H, Determine \(h_1, h_2, e_1, e_2\) (if h is not available)

\(e_1\) and \(h_1\) are the values of \(e\) and \(h\) for the unique hue having the nearest lower value of \(h\) and \(e_2\) and \(h_2\) are the values of \(e\) and \(h\) for the unique hue having the nearest higher value of \(h\).

(4) Calculate \(h\) (if necessary)

\[ h = [(H - H_1)(h_1/e_1 - h_2/e_2) - 100h_1/e_1] / [(H - H_1)(1/e_1 - 1/e_2) - 100/e_1] \]

\(H_1\) is 0, 100, 200, or 300 according to whether red, yellow, green, or blue is the hue having the nearest lower value of \(h\).

(5) Calculate \(e\)

\[ e = e_1 + (e_2 - e_1)(h - h_1)/(h_2 - h_1) \]

\(e_1\) and \(h_1\) are the values of \(e\) and \(h\) for the unique hue having the nearest lower value of \(h\) and \(e_2\) and \(h_2\) are the values of \(e\) and \(h\) for the unique hue having the nearest higher value of \(h\).

(6) Calculate C (if necessary)

\[ C = M/F_L^{0.15} \]

(7) Calculate \(s\)

\[ s = C^{1/0.69} / \left[ 2.44(J/100)^{0.67n} (1.64 - 0.29n) \right]^{1/0.69} \]
(8) Calculate a and b
\[ a = s(A/N_{bb} + 2.05)/\left[1 + (\tanh h)^2\right]^{1/2} \left\{ 50000eN_cN_{cb}/13 + s(11/23) + (108/23)(\tanh h) \right\} \]

In calculating \([1 + (\tanh h)^2]^{1/2}\) the result is taken as:
- positive for \(0^\circ \leq h < 90^\circ\)
- negative for \(90^\circ \leq h < 270^\circ\)
- positive for \(270^\circ \leq h < 360^\circ\).

\[ b = a(\tanh h) \]

(9) Calculate \(R_a', G_a',\) and \(B_a'\)
\[ R_a' = (20/61)(A/N_{bb} + 2.05) + (41/61)(11/23)a + (288/61)(1/23)b \]
\[ G_a' = (20/61)(A/N_{bb} + 2.05) - (81/61)(11/23)a - (261/61)(1/23)b \]
\[ R_a' = (20/61)(A/N_{bb} + 2.05) - (20/61)(11/23)a - (20/61)(315/23)b \]

(10) Calculate \(R', G',\) and \(B'\)
\[ R' = 100\left[(2R_a - 2)/(41 - R_a)\right]^{1/0.73} \]
\[ G' = 100\left[(2G_a - 2)/(41 - G_a)\right]^{1/0.73} \]
\[ B' = 100\left[(2B_a - 2)/(41 - B_a)\right]^{1/0.73} \]
If \(R_a' < 0\) use:
\[ R' = -100\left[2 - 2R_a\right]/(39 + R_a') \]
and similarly for the \(G'\) and \(B'\) equations.

(11) Calculate \(R_cY, G_cY,\) and \(B_cY\)
\[ R_cY = \begin{bmatrix} R'/F_L \end{bmatrix} \]
\[ G_cY = M_BM_H^{-1}\begin{bmatrix} G'/F_L \end{bmatrix} \]
\[ B_cY = \begin{bmatrix} B'/F_L \end{bmatrix} \]

(12) Calculate \(Y_c\)
\[ Y_c = 0.43231R_cY + 0.51836G_cY + 0.04929B_cY \]

(13) Calculate \((Y/Y_c)R, (Y/Y_c)G,\) and \((Y/Y_c)^{1/p}B\)
\[ (Y/Y_c)R = (Y/Y_c)R_c/\left[D(1/R_w) + 1 - D\right] \]
\[ (Y/Y_c)G = (Y/Y_c)G_c/\left[D(1/G_w) + 1 - D\right] \]
\[ (Y/Y_c)^{1/p}B = \left[(Y/Y_c)B_c\right]^{1/p}\left[D(1/B_w^p) + 1 - D\right]^{1/p} \]
If \((Y/Y_c)B_c < 0.0\) then \((Y/Y_c)^{1/p}B\) is also set to be negative.

(14) Calculate \(Y'\)
\[ Y' = 0.43231YR + 0.51836YG + 0.04929(Y/Y'_c)^{1/p}B_Y_c \]
(15) Calculate $X''$, $Y''$ and $Z''$

\[
\begin{pmatrix}
X' \\
Y' \\
Z'
\end{pmatrix} = M_b^{-1}
\begin{pmatrix}
Y_c \left(\frac{Y}{Y_c}\right)R \\
Y_c \left(\frac{Y}{Y_c}\right)G \\
Y_c \left(\frac{Y}{Y_c}\right)^{1/p} \frac{B}{\left(\frac{Y'}{Y_c}\right)^{\left(\frac{1}{p-1}\right)}}
\end{pmatrix}
\]

Note: $X''$, $Y''$, and $Z''$ are equal to the desired $X$, $Y$, and $Z$ to a very close approximation. This is because $Y'$ differs from $Y$ since $(Y/Y_c)^{1/p}BY_c$ is used instead of $YB$. However this is multiplied by 0.04929 so the difference is small.